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# Diffraction by a Semi-Infinite Screen with a Rounded End

DEMETRIOS G. MAGIROS and JOSEPH B. KELLER

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## Abstract

The diffraction of a cylindrical wave by a perfectly conducting semi-infinite thin screen with a cylindrical tip is analyzed. Three different methods are employed to find the field: the geometrical theory of diffraction, an expansion in radial eigenfunctions and the Watson transformation of the angular eigenfunction expansion. The latter two methods yield the same result, which proves that the solution can be expanded in radial eigenfunctions even though they are not complete. The asymptotic form of the solution for large ka coincides precisely with the result given by the geometrical theory of diffraction. Here $k = 2\pi/\lambda$ is the propagation constant of the field and a is the radius of the tip. This agreement proves that the geometrical theory is correct for this problem. The result determines how the field in the shadow depends upon the wavelength and the curvature of the shadow forming object[1].

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In Section 4, we prove that the radial eigenfunction expansion does represent the field u. We do this by first expanding u in angular eigenfunctions, which are known to be complete. This expansion was first given by S.N. Karp[4]. Then we convert this series into the radial eigenfunction series by applying the Watson transformation. In this way we prove that the latter series does represent u. Therefore, the result of the geometrical theory of diffraction, which is asymptotic to it, is indeed the asymptotic form of the exact solution.

The diffracted field in the shadow has been computed [1] from the geometrical solution and also from the exact solution as a function of ka. The two results agree well for $ka \geq 2$ for both boundary conditions. In the case in which $u = 0$ on the boundary, the field in the shadow is found to decrease as ka increases, being greatest when $ka = 0$, i.e. when the cylindrical tip is absent. In the other case, in which $\partial u/\partial n = 0$ on the boundary, the field in the shadow is found to oscillate as ka increases, but ultimately for sufficiently large values of ka, it also decreases monotonically. These results are interpreted in terms of the rays of the geometrical theory of diffraction.

It is to be noted that the problem under consideration is two dimensional, so that we may confine our attention to a single plane.

## 2. Geometrical solution

According to the geometrical theory of diffraction, the field u can be represented as the sum of a geometrical optics field $u_g$ and a diffracted field $u_d$. The field $u_g(P)$ at a point P is the sum of the fields on the incident and reflected rays through P. The field $u_i(P)$ on an incident ray through P is given by

$$u_i(P) = (8\pi kR)^{-1/2} e^{ikR + i\pi/4} \tag{1}$$

Here R denotes the distance from the source Q to the point P. The constants in (1) have been chosen to correspond to a source of unit strength for which the exact field would be given by $\frac{i}{4} H_0^{(1)}(kR)$. The field on a reflected ray can be constructed by a well known procedure, so we need not consider it further. Instead, we turn to the diffracted field $u_d(P)$ which is the sum of the fields on all the diffracted rays through P.

The diffracted rays are produced by the two rays from Q which are tangent to the tip of the screen. The two points of tangency are denoted by $Q_1$ and $Q_2$. (See Figure 1.) The surface ray produced at $Q_1$ will shed a diffracted ray at $P_1$ which will travel along the tangent from $P_1$ to P. It will also shed a diffracted ray at $P'_1$ which will be reflected at $P_3$ and then reach P. The surface ray itself will be reflected from the screen at $O_1$, encircle the tip and be reflected at $O_1$ again, and repeat this process infinitely many times. Each time it passes through $P_1$ and $P'_1$ in the clockwise direction, the rays it sheds will pass through P. Thus infinitely many diffracted rays will pass through P, all originally produced by the ray incident at $Q_1$. In a similar manner, the surface ray produced at $Q_2$ will be reflected at $O_1$ and then shed rays at $P_1$ and $P'_1$ which will pass through P. It will also give rise to infinitely many rays through P as it continues to travel back and forth along the surface of the tip.

As a consequence of the preceding considerations, we see that there are four types of diffracted rays which pass through a point P in the shadow. We may describe them as follows:

I Rays produced at $Q_1$ and diffracted at $P_1$. The ray $QQ_1P_1P$ is the simplest ray of this type. For each non-negative value of the integer n, there is one of these rays which makes n trips around the circle. This ray is reflected 2n times at $O_1$. Now the reflection coefficient R has the value -1 for the case in which $u = 0$ on the screen and +1 for the case in which $\partial u/\partial n = 0$ on the screen. Thus in either case $R^{2n} = 1$. The path length of such a ray along the circle is $4\pi an + t$ where $t = Q_1P_1$.

II Rays produced at $Q_1$ and diffracted at $P'_1$. The ray $QQ_1P_1P'_1P_3P$ is the simplest ray of this type. For each $n \geq 0$, the ray which makes n round trips is reflected $2n + 1$ times, including the reflection at $P_3$. Thus $R^{2n+1} = -1$ if $u = 0$ on the screen and $R^{2n+1} = 1$ if $\partial u/\partial n = 0$. The path length of such a ray along the circle is $4\pi an + t + t_1$ where $t_1 = P_1P'_1$.

III Rays produced at $Q_2$ and diffracted at $P_1$. The ray $QQ_2O_1Q_2Q_1P_1P$ is the simplest ray of this type. For each $n \geq 0$, the ray which makes n round trips is reflected $2n + 1$ times. Its path length on the circle is $4\pi an + t_2$ where $t_2 = Q_2O_1 + O_1Q_2Q_1P_1$.

IV Rays produced at $Q_2$ and diffracted at $P'_1$. The ray $QQ_2O_1Q_2Q_1P_1P'_1P_3P$ is the simplest ray of this type. The ray which makes $n \geq 0$ round trips is reflected $2n + 2$ times, including the reflection at $P_3$. Its path length along the circle is $4\pi an + t_2 + t_1$.

According to equation (10a) of reference [3], the field on a diffracted ray of type I is given by

$$(2) \qquad u_i(Q_1) \; \frac{e^{iks}}{s^{1/2}} \sum_m D_m^{\,2} \, e^{(ik-\alpha_m)T_n}$$

In (2), s denotes distance along the diffracted ray from the point where it leaves the surface, $T_n = 4\pi an + t$ denotes its arclength along the surface, and $\alpha_m$ and $D_m$ are certain constants. The field on the rays of other types is also given by (2) with the appropriate values of s and $T_n$ in each case, and with an appropriate power of the reflection coefficient R.

Let the coordinates of Q be $(r_o, \theta_o)$ and those of P be $(r, \theta)$ in a polar coordinate system with its origin at the center of the tip and the polar axis along the screen. Then $R = (r_o^{\,2} - a^2)^{1/2}$ and $s = (r^2 - a^2)^{1/2}$. We insert these values into (2) using (1) for $u_i(Q_1)$, and we sum over n. This summation merely involves a geometric series, which can be summed explicitly. Then we add together the corresponding sums for the four types of rays and obtain the diffracted field at a point P in the shadow

$$(3) \quad u_d(P) = \frac{i(8\pi k)^{-1/2} \exp\left[ik(r_o^{\,2}-a^2)^{1/2} + ik(r^2-a^2)^{1/2} - i\pi/4\right]}{(r_o^{\,2}-a^2)^{1/4}(r^2-a^2)^{1/4}}$$

$$\sum_m \frac{D_m^{\,2}}{1-e^{4\pi a(ik-\alpha_m)}} \left[ e^{(ik-\alpha_m)t} \mp e^{(ik-\alpha_m)(t+t_1)} \mp e^{(ik-\alpha_m)t_2} + e^{(ik-\alpha_m)(t_2+t_1}\right.$$

In (3) the upper signs apply in case $u = 0$ on the screen while the lower signs apply if $\partial u/\partial n = 0$ on the screen.

In terms of the coordinates of Q and P, the distances $t$, $t_1$ and $t_2$ are given by

$$t = a\left[\theta - \theta_o - \cos^{-1}(a/r_o) - \cos^{-1}(a/r)\right]$$

$$t_1 = 2a(2\pi - \theta) \tag{4}$$

$$t_2 = a\left[\theta + \theta_o - \cos^{-1}(a/r_o) - \cos^{-1}(a/r)\right] .$$

The diffraction coefficients $D_m$ and the decay constants $\alpha_m$ are given by [3], [5]

$$\alpha_m = e^{-\pi i/6} \left(\frac{k}{6a^2}\right)^{1/3} q_m \tag{5}$$

$$D_m^2 = \left[\frac{\pi^9}{2^5\, 3^8}\, \frac{a^2}{k}\right]^{1/6} e^{\pi i/12} \left[A'(q_m)\right]^{-2} , \tag{6a}$$

$$D_m^2 = \left[\frac{\pi^9}{2^5\, 3^2}\, \frac{a^2}{k}\right]^{1/6} e^{\pi i/12}\, q_m^{-1} \left[A(q_m)\right]^{-2} . \tag{6b}$$

In these equations $A(q)$ denotes the Airy function. For the boundary condition $u = 0$, $q_m$ is the $m^{th}$ positive zero of $A(q)$ while for the boundary condition $\partial u/\partial n = 0$, $q_m$ is the $m^{th}$ positive zero of $A'(q)$.

We now insert the preceding results into (3) and obtain in the cases $u = 0$ and $\partial u/\partial n = 0$ the respective results for a point P in the shadow

$$(7a)\quad u_d(P) = \frac{\pi}{6}\left(\frac{a}{6k^2}\right)^{1/3} \frac{\exp\left[ik(r_o^2-a^2)^{1/2} + ik(r^2-a^2)^{1/2} + 5\pi i/6\right]}{(r_o^2-a^2)^{1/4}\,(r^2-a^2)^{1/4}}$$

$$\sum_m \frac{\sin\left[(ka+i\alpha_m a)(2\pi-\theta)\right]\,\sin\left[(ka+i\alpha_m a)\theta_o\right]}{\left[A'(q_m)\right]^2\,\sin\left[(ka+i\alpha_m a)2\pi\right]}\,\exp\left\{-i(ka+i\alpha_m a)\left[\cos^{-1}\frac{a}{r_o} + \cos^{-1}\frac{a}{r}\right]\right\}$$

$$(7b)\quad u_d(P) = \frac{\pi}{18}\left(\frac{a}{6k^2}\right)^{1/3} \frac{\exp\left[ik(r_o^2-a^2)^{1/2} + ik(r^2-a^2)^{1/2} + 5\pi i/6\right]}{(r_o^2-a^2)^{1/4}\,(r^2-a^2)^{1/4}}$$

$$\sum_m \frac{\cos\left[(ka+i\alpha_m a)(2\pi-\theta)\right]\,\cos\left[(ka+i\alpha_m a)\theta_o\right]}{q_m\,A^2(q_m)\,\sin\left[(ka+i\alpha_m a)2\pi\right]}\,\exp\left\{-i(ka+i\alpha_m a)\left[\cos^{-1}\frac{a}{r_o} + \cos^{-1}\frac{a}{r}\right]\right\}$$

These results hold when $\theta > \theta_o$. When $\theta < \theta_o$ we obtain the same results with $\theta$ and $\theta_o$ interchanged. Slightly different results can be obtained in the same way for a point P in the illuminated region.

## 3. Radial eigenfunction expansion

We shall now formulate a boundary value problem for the total field u and solve it by the method of expansion in radial eigenfunctions. The conditions to be satisfied by $u(r,\theta)$ are (1), (2) and (3a) or (3b)

$$(\Delta + k^2)u = r^{-1}\,\delta(r-r_o)\,\delta(\theta-\theta_o) \tag{1}$$

$$\lim_{r\to\infty} r^{1/2}(u_r - iku) = 0 \tag{2}$$

$$u\,(a,\theta) = u\,(r,0) = u\,(r,2\pi) = 0 \tag{3a}$$

$$u_r(a,\theta) = u_\theta(r,0) = u_\theta(r,2\pi) = 0\ . \tag{3b}$$

In order to construct u we introduce the radial eigenfunctions $H^{(1)}_{\nu_m}(kr)$. These Hankel functions are the radial factors of product solutions of the homogeneous equation corresponding to (1). They also satisfy the radiation condition (2). The eigenvalues $\nu_m$ are determined by requiring these functions to satisfy the first of the boundary conditions (3a) or (3b) which become respectively

$$H^{(1)}_{\nu_m}(ka) = 0 \tag{4a}$$

$$H^{(1)'}_{\nu_m}(ka) = 0\ . \tag{4b}$$

For large values of ka, and fixed m, the roots of (4a) and of (4b) are given by

$$(5) \qquad \nu_m = ka + q_m \, (ka/6)^{1/3} \, e^{i\pi/3} + 0\left[(ka)^{-1/3}\right] .$$

In the case of (4a), $q_m$ is the $m^{th}$ positive zero of the Airy function $A(q)$ while in the case of (4b) $q_m$ is the $m^{th}$ positive zero of $A'(q)$. We now assume that u can be represented by the series

$$(6) \qquad u = \sum_{m=1}^{\infty} f_m(\theta) \, H^{(1)}_{\nu_m} (kr) .$$

To determine the coefficient functions $f_m(\theta)$ we insert (6) into (1) and make use of Bessel's equation to obtain

$$(7) \qquad \sum_{m=1}^{\infty} r^{-2} \, H^{(1)}_{\nu_m} (kr) \left[ f_m''(\theta) + \nu_m^2 \, f_m(\theta) \right] = r^{-1} \, \delta(r-r_o) \, \delta(\theta-\theta_o) .$$

It is easy to prove that the radial eigenfunctions $H^{(1)}_{\nu_m}(kr)$ are mutually orthogonal, which may be expressed in the form

$$(8) \qquad \int_a^{\infty} r^{-1} \, H^{(1)}_{\nu_m}(kr) \, H^{(1)}_{\nu_n}(kr) \, dr = N_n \, \delta_{mn} .$$

An analysis similar to that of Sommerfeld [6] yields for the constant $N_n$

the following results in the respective cases u = 0 and $\partial u/\partial n = 0$ on the screen

(9a)
$$N_n = \left[ \frac{2i}{\pi\nu} \frac{\frac{\partial}{\partial\nu} H_\nu^{(1)}(ka)}{H_\nu^{(2)}(ka)} \right]_{\nu = \nu_n}$$

(9b)
$$N_n = \left[ \frac{2i}{\pi\nu} \frac{\frac{\partial}{\partial\nu} H_\nu^{(1)'}(ka)}{H_\nu^{(2)'}(ka)} \right]_{\nu = \nu_n}$$

We now multiply (7) by $r^{-1} H_{\nu_n}^{(1)}(kr)$ and integrate the resulting equation from a to $\infty$ with respect to r. Then by using (8) we obtain

(10)
$$f_n''(\theta) + \nu_n^2 f_n(\theta) = N_n^{-1} H_{\nu_n}^{(1)}(kr_o)\, \delta(\theta-\theta_o) \cdot$$

In order to determine $f_n(\theta)$ from (10), we must impose the second and third of the boundary conditions (3a) or (3b) on u, which is given by (6). By again utilizing the orthogonality condition (8) these boundary conditions yield, in the respective cases

(11a)
$$f_n(0) = f_n(2\pi) = 0$$

(11b)
$$f_n'(0) = f_n'(2\pi) = 0 \,.$$

The solutions of (10) which satisfy (11a) or (11b), respectively, are

$$(12a) \qquad f_n(\theta) = -\nu_n^{-1} \csc 2\pi\nu_n \, N_n^{-1} \, H^{(1)}_{\nu_n}(kr_o) \sin \nu_n\theta_< \sin \nu_n(2\pi-\theta_>)$$

$$(12b) \qquad f_n(\theta) = +\nu_n^{-1} \csc 2\pi\nu_n \, N_n^{-1} \, H^{(1)}_{\nu_n}(kr_o) \cos \nu_n\theta_< \cos \nu_n(2\pi-\theta_>).$$

In (12a) and (12b) $\theta_<$ and $\theta_>$ denote respectively the smaller and larger of the two angles $\theta$ and $\theta_o$, both of which lie between 0 and $2\pi$.

We now insert (12a) into (6), and use (9a) for $N_n$. Thus we obtain for the solution u in the case of the boundary condition $u = 0$, the result

$$(13a) \qquad u = +\frac{i\pi}{2} \sum_{n=1}^{\infty} H^{(1)}_{\nu_n}(kr_o) H^{(1)}_{\nu_n}(kr) \frac{\sin\nu_n\theta_<}{\sin 2\pi\nu_n} \sin\nu_n(2\pi-\theta_>) \frac{H^{(2)}_{\nu_n}(ka)}{\frac{\partial}{\partial\nu} H^{(1)}_{\nu_n}(ka)} .$$

In the case of the boundary condition $\partial u/\partial n = 0$ we obtain instead, from (12b), (9b) and (6), the result

$$(13b) \qquad u = -\frac{i\pi}{2} \sum_{n=1}^{\infty} H^{(1)}_{\nu_n}(kr_o) \, H^{(1)}_{\nu_n}(kr) \frac{\cos\nu_n(2\pi-\theta_>)}{\sin 2\pi\nu_n} \cos\nu_n\theta_< \frac{H^{(2)\prime}_{\nu_n}(ka)}{\frac{\partial}{\partial\nu} H^{(1)\prime}_{\nu_n}(ka)} .$$

Let us now expand these expressions asymptotically for large values of ka. If $r > a$ and $r_o > a$, we may use the Debye asymptotic expansions of the Hankel functions with arguments $kr$ and $kr_o$. However, for the Hankel functions with argument ka, which is nearly equal to the order $\nu_n$, we must use the expansion in terms of Airy functions. When we insert these expansions into (13a) and (13b), we obtain the results

$$(14a) \qquad u(r,\theta) \sim \frac{\pi}{6^{4/3}} (ka)^{1/3} \sum_{n=1}^{\infty} \frac{\exp\left[i\sqrt{k^2r^2-\nu_n^2} + i\sqrt{k^2r_o^2-\nu_n^2} + \frac{5\pi i}{6}\right]}{(k^2r^2-\nu_n^2)^{1/4}(k^2r_o^2-\nu_n^2)^{1/4}}$$

$$\frac{\sin\nu_n\theta_< \sin\nu_n(2\pi-\theta_>)}{\left[A'(q_n)\right]^2 \sin 2\pi\nu_n} \exp\left[-i\nu_n\left[\cos^{-1}\left(\frac{\nu_n}{kr}\right)+\cos^{-1}\left(\frac{\nu_n}{kr_o}\right)\right]\right] .$$

$$(14b) \qquad u(r,\theta) \sim \frac{\pi}{3\cdot 6^{4/3}} (ka)^{1/3} \sum_{n=1}^{\infty} \frac{\exp\left[i\sqrt{k^2r^2-\nu_n^2} + i\sqrt{k^2r_o^2-\nu_n^2} + \frac{5\pi i}{6}\right]}{(k^2r^2-\nu_n^2)^{1/4}\,(k^2r_o^2-\nu_n^2)^{1/4}}$$

$$\frac{\cos\nu_n\theta_< \cos\nu_n(2\pi-\theta_>)}{q_n\, A^2(q_n) \sin 2\pi\nu_n} \exp\left[-i\nu_n\left[\cos^{-1}\left(\frac{\nu_n}{kr}\right) + \cos^{-1}\left(\frac{\nu_n}{kr_o}\right)\right]\right] .$$

By using (5) for $\nu_n$ in these equations, we can simplify them to the form

$$(15a) \qquad u(r,\theta) \sim \frac{\pi}{6^{4/3}} \frac{(k^{-2}a)^{1/3} e^{(5\pi i)/6}}{(r^2-a^2)^{1/2}(r_o^2-a^2)^{1/2}} \exp\left[ik\left[(r^2-a^2)^{1/2}+(r_o^2-a^2)^{1/2}\right]\right]$$

$$\sum_{n=1}^{\infty} \frac{\sin \nu_n\theta_< \sin \nu_n(2\pi-\theta_>)}{\left[A'(q_n)\right]^2 \sin 2\pi\nu_n} \exp\left[-i\nu_n\left(\cos^{-1}\frac{a}{r} + \cos^{-1}\frac{a}{r_o}\right)\right]$$

$$\text{(15b)} \qquad u(r,\theta) \sim \frac{\pi}{3\cdot 6^{4/3}} \frac{(k^{-2}a)^{1/3} e^{(5\pi i)/6}}{(r^2-a^2)^{1/2}(r_o^2-a^2)^{1/2}} \exp\left[ik\left[(r^2-a^2)^{1/2}+(r_o^2-a^2)^{1/2}\right]\right]$$

$$\sum_{n=1}^{\infty} \frac{\cos \nu_n\theta_< \cos \nu_n(2\pi-\theta_>)}{q_n A^2(q_n)\sin 2\pi\nu_n} \exp\left[-i\nu_n\left(\cos^{-1}\frac{a}{r} + \cos^{-1}\frac{a}{r_o}\right)\right] .$$

The results (15) coincide exactly with the results (7) of section 2 which were obtained by the geometrical theory of diffraction for a point in the shadow. This verifies that the geometrical theory is correct in this case since (13), from which (15) was obtained, does indeed represent the solution as we shall show in the next section. However the asymptotic evaluation of (13), which led to (15), is correct only for a finite number of terms in the series. This is so because in the later terms $\nu_n$ becomes large compared to ka, kr and $kr_o$ and this was not taken into account in the asymptotic evaluation. Furthermore for n larg $\nu_n$ is not given by (5). But as we shall see, (15) converges very rapidly for points in the shadow so the error it contains in the later terms is asymptoticall negligible. For points in the illuminated region however, (15) does not converge although (13) does. Therefore a different asymptotic evaluation must be made in this region. This will be done in the next section.

## 4. Angular eigenfunction expansion.

The eigenvalue problem for the radial eigenfunctions involves the radiation condition and, therefore, it is not of Sturm-Liouville type. Consequently, we do not know whether these eigenfunctions are complete. Therefore, we cannot be sure that the solution u can be represented by the series (6) of Section 3. We shall now prove that it can be so represented and that it is given by (13a) or (13b).

We begin with a representation of u in terms of the angular eigenfunctions which are complete, since they are solutions of a Sturm-Liouville problem. This representation was given by S.N. Karp[4]. By modifying his result to correspond to our problem (1)-(3) of section 3, or by direct expansion of u in angular eigenfunctions, we find in the two cases

$$(1a) \qquad u = \frac{i}{2}\sum_{n=1}^{\infty} H^{(1)}_{n/2}(kr_>)\left[J_{n/2}(kr_<) - \frac{J_{n/2}(ka)}{H^{(1)}_{n/2}(ka)}\, H^{(1)}_{n/2}(kr_<)\right]\sin\frac{n}{2}\theta_< \sin\frac{n}{2}\theta_>$$

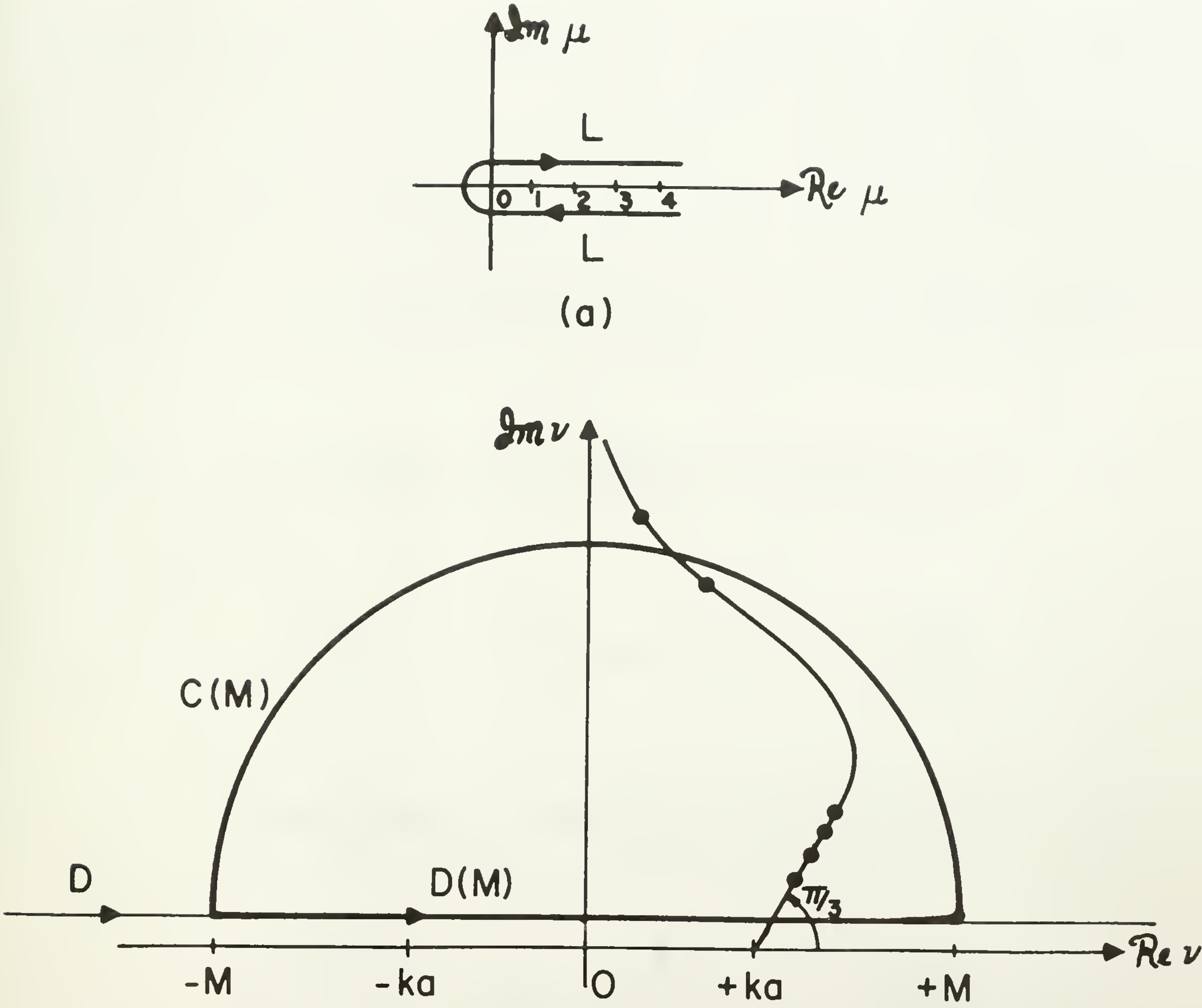
$$(1b) \qquad u = -\frac{i}{2}\sum_{n=1}^{\infty} H^{(1)}_{n/2}(kr_>)\left[J_{n/2}(kr_<) - \frac{J'_{n/2}(ka)}{H^{(1)\prime}_{n/2}(ka)}\, H^{(1)}_{n/2}(kr_<)\right]\sin\frac{n}{2}\theta_< \sin\frac{n}{2}\theta_>$$

These series are useful for small values of ka but not for large values of ka.

To obtain representations of u useful for large ka we apply the Watson transformation to the series. First we use in (1a) and (1b) the identities

$$\sin\frac{n}{2}\theta_> = -\sin\frac{n}{2}(2\pi - \theta_>)/\cos n\pi$$

$$\cos\frac{n}{2}\theta_> = \cos\frac{n}{2}(2\pi - \theta_>)/\cos n\pi$$



(a)

(b)

Figure 2 - a - The contour L in the μ plane encircles the positive real axis in the clockwise direction.

b - The contour D is shown just above the real axis of the ν plane. The segment D(M) of it extends from Re ν = -M to Re ν = M. The semicircle C(M) into which D(M) is deformed, is also shown. The poles of the integrand in the half plane Im ν > 0 lie on the curved line shown in the first quadrant.

Then we represent each series as a contour integral. This is accomplished by first choosing a contour L in the $\mu$ plane which encircles the non-negative integers $\mu = n$, $n = 0,1,2,\ldots$ . Then an integrand must be chosen which has as its only singularities within L poles at these integers. In addition for each n, the residue at the n-th pole must be the n-th term in the series. In this way we arrive at the following integral representations

$$\text{(2a)}\qquad u = \frac{1}{4}\int_L \frac{1}{\sin\mu\pi}\left[J_{\mu/2}(kr_<)H^{(1)}_{\mu/2}(ka) - J_{\mu/2}(ka)H^{(1)}_{\mu/2}(kr_<)\right] \frac{H^{(1)}_{\mu/2}(kr_>)}{H^{(1)}_{\mu/2}(ka)} \sin\frac{\mu\theta_<}{2} \sin\frac{\mu}{2}(2\pi - \theta_>)d\mu$$

$$\text{(2b)}\qquad u = \frac{1}{4}\int_L \frac{1}{\sin\mu\pi}\left[J_{\mu/2}(kr_<)H^{(1)\prime}_{\mu/2}(ka) - J'_{\mu/2}(ka)H^{(1)}_{\mu/2}(kr_<)\right] \frac{H^{(1)}_{\mu/2}(kr_>)}{H^{(1)\prime}_{\mu/2}(ka)} \cos\frac{\mu\theta_<}{2} \cos\frac{\mu}{2}(2\pi - \theta_>)d\mu \ .$$

The contour L encircles the positive real axis of the $\mu$ plane in the clockwise direction as is shown in Figure 2.

Let us replace $\mu$ by $\nu = \mu/2$ in (2). Let us also change $\nu$ to $-\nu$ in that part of the integral for which $\mathrm{Im}\nu < 0$. Then that part of the contour is transformed into a path from $-\infty$ to 0 lying just above the negative real axis. The integrand is unchanged as we see from the relations

$$(3)\qquad \begin{aligned} H^{(1)}_{-\nu} &= e^{i\pi\nu}\ H^{(1)}_{\nu} \\ H^{(2)}_{-\nu} &= e^{i\pi\nu}\ H^{(2)}_{\nu} \ . \end{aligned}$$

Thus the contour L can be replaced by the contour D which is a line from $-\infty$ to $\infty$ lying just above the real axis of the $\nu$ plane and (2) becomes

$$(4a)\qquad u = \frac{1}{2} \int_D \frac{1}{\sin 2\pi\nu} \left[ J_\nu(kr_<) H^{(1)}_\nu(ka) - J_\nu(ka) H^{(1)}_\nu(kr_<) \right] \frac{H^{(1)}_\nu(kr_>)}{H^{(1)}_\nu(ka)} \sin \nu\theta_< \sin \nu(2\pi - \theta_>) d\nu$$

$$(4b)\qquad u = \frac{1}{2} \int_D \frac{1}{\sin 2\pi\nu} \left[ J_\nu(kr_<) H^{(1)\prime}_\nu(ka) - J^\prime_\nu(ka) H^{(1)}_\nu(kr_<) \right] \frac{H^{(1)}_\nu(kr_>)}{H^{(1)\prime}_\nu(ka)} \cos \nu\theta_< \cos \nu(2\pi - \theta_>) d\nu \ .$$

Let D(M) denote that part of the contour from -M to +M just above the real axis (see Fig. 2). Let C(M) denote a semi-circle in the upper half plane with radius M and center at the origin. We wish to deform the partial contour D(M) into C(M). This is permissible since the integrands in (4) are analytic functions of $\nu$. They are regular in the upper half of the $\nu$ plane except for poles at the zeroes of their denominators. These zeroes are the values of $\nu$ for which either

(5a) $$H_{\nu}^{(1)}(ka) = 0$$

or

(5b) $$H_{\nu}^{(1)'}(ka) = 0 .$$

In each case we shall denote these values of $\nu$ by $\nu_n$, $n = 1,2,\ldots$ . Let $N(M)$ denote the number of these values whose distance from the origin is less than $M$ and let $R_n$ denote the residue of the integrand at $\nu_n$. Then we have in each case, omitting the integrands, and choosing $M$ so that no zero lies on $C(M)$,

(6) $$\int_{D(M)} = \sum_{n=1}^{N(M)} R_n + \int_{C(M)} .$$

From (4) and (6) we have in each case

(7) $$u = \lim_{M\to\infty} \int_{D(M)} = \sum_{n=1}^{\infty} R_n + \lim_{M\to\infty} \int_{C(M)} .$$

We shall now assume that the limit of the integral in (7) is zero. Then (7) yields for $u$ the following series, in which the residues $R_n$ have been written explicitly

(8a) $$u(r,\theta) = \frac{i\pi}{2} \sum_{n=1}^{\infty} H_{\nu_n}^{(1)}(kr_<) H_{\nu_n}^{(1)}(kr_>) \frac{\sin \nu_n \theta_<}{\sin 2\pi\nu_m} \sin \nu_n (2\pi - \theta_>) \frac{H_{\nu_n}^{(2)}(ka)}{\frac{\partial}{\partial\nu} H_{\nu_n}^{(1)}(ka)}$$

(8b) $$u(r,\theta) = -\frac{i\pi}{2} \sum_{n=1}^{\infty} H_{\nu_n}^{(1)}(kr_<) H_{\nu_n}^{(1)}(kr_>) \frac{\cos \nu_n \theta_<}{\sin 2\pi\nu_n} \cos \nu_n (2\pi - \theta_>) \frac{H_{\nu_n}^{(2)'}(ka)}{\frac{\partial}{\partial\nu} H_{\nu_n}^{(2)'}(ka)}$$

These series are identical with the radial eigenfunction expansions of u given by (13) of section 3. Thus the validity of those expansions is proved.

Another representation of u can be obtained by using in (4) the identities

(9a) $$\sin \nu(2\pi-\theta_>) = -e^{i2\pi\nu}\sin \nu\theta_> + e^{i\nu\theta_>}\sin 2\pi\nu$$

(9b) $$\cos \nu(2\pi-\theta_>) = e^{i2\pi\nu}\cos \nu\theta_> - ie^{i\nu\theta_>}\sin 2\pi\nu \ .$$

The resulting integrands contain one term proportional to $e^{i\nu\theta_>}$ and another proportional to $\sin \nu\theta_>$ or $\cos \nu\theta_>$. The integral of the former term is kept as an integral over D while the integral of the latter term is converted into a residue series as before. In this way we obtain the alternative representations

(10a) $$u = \frac{i\pi}{2}\sum_{n=1}^{\infty} H^{(1)}_{\nu_n}(kr_<)H^{(1)}_{\nu_n}(kr_>)\,\frac{\sin \nu_n\theta_< \sin \nu_n\theta_>}{\sin 2\pi\nu_n}\,e^{i2\pi\nu_n}\,\frac{H^{(2)}_{\nu_n}(ka)}{\frac{\partial}{\partial\nu}H^{(1)}_{\nu_n}(ka)}$$

$$-\frac{1}{4}\int_D H^{(1)}_{\nu}(kr_<)H^{(1)}_{\nu}(kr_>)\sin \nu\theta_<\, e^{i\nu\theta_>}\,\frac{H^{(2)}_{\nu}(ka)}{H^{(1)}_{\nu}(ka)}\,d\nu$$

(10b) $$u = \frac{-i\pi}{2}\sum_{n=1}^{\infty} H^{(1)}_{\nu_n}(kr_<)H^{(1)}_{\nu_n}(kr_>)\,\frac{\cos \nu_n\theta_< \cos \nu_n\theta_>}{\sin 2\pi\nu_n}\,e^{i2\pi\nu_n}\,\frac{H^{(2)\prime}_{\nu_n}(ka)}{\frac{\partial}{\partial\nu}H^{(1)\prime}_{\nu_n}(ka)}$$

$$+\frac{i}{4}\int_D H^{(1)}_{\nu}(kr_<)H^{(1)}_{\nu}(kr_>)\cos \nu\theta_<\, e^{i\nu\theta_>}\,\frac{H^{(2)\prime}_{\nu}(ka)}{H^{(1)\prime}_{\nu}(ka)}\,d\nu \ .$$

We shall see in the next section that this representation is more useful in the illuminated region while (8) is more useful in the shadow region.

## 5. Convergence of the series.

To investigate the convergence of the various series, we follow the procedure of W. Franz [7]. The Hankel function $H_\nu^{(1)}(x)$ of large complex order $\nu$ and fixed real argument x has the asymptotic form [7]

$$(1) \qquad H_\nu^{(1)}(x) \sim -2\sqrt{\frac{2}{\pi\nu}} \sinh\left[\nu \log\frac{2\nu}{x} - \nu + i\frac{\pi}{4}\right] .$$

Consequently the values of $\nu$ at which $H_\nu^{(1)}(x)$ vanishes are asymptotically determined by

$$(2) \qquad \nu\left(\log\frac{2\nu}{x} - 1\right) = \left(n - \frac{1}{4}\right) i\pi .$$

In (2) n denotes a large integer. To solve (2) we set $\nu = \rho e^{i\phi}$. Then since the real part of the left side of (2) must be zero we find that for large $\rho$, $\phi$ and $\rho$ must be related by

$$(3) \qquad \phi \sim \frac{\pi}{2}\left(1 - \frac{1}{\log 2\rho/x}\right) .$$

From the imaginary parts in (2) we then find, denoting $\rho$ by $\rho_n$,

$$(4) \qquad \rho_n \sim \frac{(n-\frac{1}{4})\pi}{\log\frac{2(n-\frac{1}{4})\pi}{x}}$$

Thus for large positive n the zeros $\nu_n$ of $H_\nu^{(1)}(x)$ are

$$(5) \qquad \nu_n \sim \frac{(n-\frac{1}{4})\pi}{\log\frac{2(n-\frac{1}{4})\pi}{x}} \exp\left[i\frac{\pi}{2}\left(1 - \frac{1}{\log\left[\frac{2(n-\frac{1}{4})\pi}{x\log\frac{2(n-\frac{1}{4})\pi}{x}}\right]}\right)\right]$$

From (1) we have

$$(6) \qquad H^{(1)\prime}_{\nu}(x) \sim \frac{2}{x} \sqrt{\frac{2\nu}{\pi}} \cosh\left[\nu \log \frac{2\nu}{x} - \nu + \frac{i\pi}{4}\right] .$$

Then (2)-(5) hold for the zeroes of $H^{(1)\prime}_{\nu}(x)$ if n is replaced by $n + \frac{1}{2}$.

By using (1) and (5) with $x = ka$ we find

$$(7) \qquad |H^{(1)}_{\nu_n}(kr_>)| \sim \sqrt{\frac{2}{\pi\nu_n}} \exp\left[\rho_n \frac{\pi}{2} \frac{\log(a/r_>)}{\log(2\rho_n/ka)}\right] .$$

To evaluate $H^{(2)}_{\nu_n}(ka)$ we use the relation $H^{(2)}_{\nu}(x) = \overline{H^{(1)}_{\bar{\nu}}(x)}$ valid for x real. Then we see that (1) holds for $H^{(2)}_{\nu}(x)$ with $i\pi/4$ replaced by $-i\pi/4$. Therefore we have

$$(8) \qquad H^{(2)}_{\nu_n}(ka) \sim 2 \sqrt{\frac{2}{\pi\nu_n}} \, (-1)^n$$

From (1) we also obtain

$$(9) \qquad \frac{\partial}{\partial\nu} H^{(1)}_{\nu_n}(ka) \sim -2 \sqrt{\frac{2}{\pi\nu_n}} \log\left(\frac{2\nu_n}{ka}\right) \cdot (-1)^n$$

Upon combining (7)-(9) we obtain for the asymptotic form of the absolute value of the n-th term in (8a) of section 4 the expression

$$(10) \qquad \frac{1}{2\rho_n \log \frac{\rho_n}{ka}} \exp\left(-\rho_n\left[\theta_> - \theta_< - \frac{\pi}{2} \frac{\log \frac{a^2}{r_< r_>}}{\log \frac{2\rho_n}{ka}}\right]\right) .$$

This expression diminishes sufficiently rapidly as n increases to guarantee that the series (8a) of section 4 converges absolutely for $\theta_> > \theta_<$ , i.e. for $\theta \neq \theta_o$. The same result applies to the series (8b) of section 4.

We shall now examine the leading terms in these series in order to show how rapidly they diminish. For any fixed value of n and ka large, $\nu_n$ is asymptotically given by (5) of section 3. Therefore for fixed n the order and argument of the Hankel functions in the series (8) of section 4 are both large. Thus for large ka we may evaluate those Hankel functions with argument $kr_<$ or $kr_>$ by using the Debye asymptotic form which is

$$(11) \qquad H_\nu^{(1)}(x) \sim \sqrt{\frac{2}{\pi}}\,(x^2-\nu^2)^{-1/4}\,\exp\left[i(x^2-\nu^2)^{1/2}-i\nu\cos^{-1}(\nu/x)-i\pi/4\right] .$$

For the Hankel functions of argument ka we must use instead the Airy function representation

$$(12) \qquad H_\nu^{(1)}(x) \sim \frac{2}{\pi}e^{-i\pi/3}\left(\frac{6}{x}\right)^{1/3}A\left[\left(\frac{6}{x}\right)^{1/3}e^{-i\pi/3}(\nu-x)\right] .$$

By using these expansions in (8) of section 4 we obtain the series (15) of section 3. If we further retain only the leading terms in the trigonometric factors in (15a) the summand becomes

$$(13) \qquad \frac{1}{2i\left[A'(q_n)\right]^2}\,\exp\left\{i\nu_n\left[\theta_>-\theta_<-\cos^{-1}(a/r)-\cos^{-1}(a/r_o)\right]\right\} .$$

The absolute value of (13) is asymptotically

$$(14) \qquad \frac{1}{2\left[A'(q_n)\right]^2}\,\exp\left\{-q_n(ka/6)^{1/3}\sin\frac{\pi}{3}\left[\theta_>-\theta_<-\cos^{-1}\left(\frac{a}{r}\right)-\cos^{-1}\left(\frac{a}{r_o}\right)\right]\right\} .$$

The $q_n$ are positive and increase with n like $n^{2/3}$. Therefore the absolute value (14) decreases with increasing n provided that

$$(15) \qquad \theta_{>} - \theta_{<} - \cos^{-1}(\frac{a}{r}) - \cos^{-1}(\frac{a}{r_o}) > 0 \; .$$

For a point in the shadow region B shown in Fig. 1, $\theta_{>} = \theta_o$, $\theta_{<} = \theta$ and

$$(16) \qquad \theta_o > \theta + \cos^{-1}(\frac{a}{r}) + \cos^{-1}(\frac{a}{r_o}) \; .$$

Therefore (15) is satisfied for points in B. Similarly for points in the shadow region A of Fig. 1, $\theta_{>} = \theta$, $\theta_{<} = \theta$ and

$$(17) \qquad \theta > \theta_o + \cos^{-1}(\frac{a}{r}) + \cos^{-1}(\frac{a}{r_o}) \; .$$

Thus (15) is also satisfied for points in A. We conclude that (15) is satisfied for points in the shadow region but not for points in the illuminated region. The same results apply to the series (15b). Consequently for points in the shadow the terms in the radial eigenfunction series (13) of section 3 or (8) of section 4 decrease rapidly like $\exp[-n^{2/3}(ka)^{1/3}c]$ where c is a positive constant. Thus the radial eigenfunction series converges rapidly only in the shadow although it also converges in the illuminated region. The series (7) of section 2, which results from the geometrical theory of diffraction for points in the shadow, also converges rapidly. In the shadow it coincides with the asymptotic expansion (15) of the radial eigenfunction series. Outside the shadow the series (15) diverges. The representation (10) of section 4 is useful in the illuminated region since the series in it converge rapidly there as can be shown by an analysis like that we have just given. In fact the asymptotic expansion of the series in (10) of section 4 yields exactly the same result as that given by the geometrical theory of diffraction for points in the illuminated region. The asymptotic evaluation of the integrals in (10) yields the incident and reflected fields. However we shall not carry out these calculations.

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